Then ΣW = (Area of A-rectangle) $\times x^2 \times \gamma$ if the scale of plotting is 1 $cm = x$ meters and γ is the unit weight of soil.

The same procedure can be adopted for ΣN and ΣT. If N_1, N_2... and T_1, T_2 .. are the normal and tangential components of Z_1, Z_2.... as shown in Fig. 13.10 (*a*), the N-rectangle and T-rectangle are constructed as shown in Fig.13.10 (*c*) and (*d*), and their areas A_N and A_T are calculated. Then we have

$$\Sigma N = A_N . x^2 . \gamma$$

$$\Sigma T = A_T . x^2 . \gamma$$

13.6 Factors of Safety Used in Stability Analysis of Slopes

1. Factor of safety with respect to cohesion assuming friction to be fully mobilised, is given by

$$F_c = \frac{c}{c_m} \quad \text{.........(i)}$$

where c = ultimate cohesion

c_m = mobilised cohesion

2. Factor of safety with respect to friction assuming cohesion to be fully mobilised is given by

$$F_\phi = \frac{\tan\phi}{\tan\phi_m} \approx \frac{\phi}{\phi_m} \quad \text{.........(ii)}$$

where ϕ = ultimate angle of shearing resistance

ϕ_m = mobilised angle of shearing resistance

3. Factor of safety with respect to shear strength is given by

$$F = \frac{\tau_f}{\tau} \quad \text{.........(iii)}$$

where ultimate shear strength, $\tau_f = c + \sigma \tan\phi$

and mobilised shear strength $\tau = c_m + \sigma \tan \phi_m$

4. Factor of safety with respect to height is given by

$$F_H = \frac{H_c}{H}$$

where H_c = critical height of slope

H = actual height of slope.

Also $F_H = F_C$, assuming cohesion to be fully mobilised.

13.7 Friction Circle Method

In the friction circle method the slip surface is assumed to be cylindrical *i.e.*, arc of a circle in section. The sliding soil mass is assumed to be acted upon by three forces keeping it in equilibrium, as shown in Fig. 13.11 (*a*)

(*i*) The weight, W, of the sliding soil mass $ABDA$, acting vertically through its centre of gravity,

(*ii*) The resultant cohesive force, $c_m \bar{L}$, acting parallel to chord AD and at distance a from centre of rotation O, where $a = r.\frac{\hat{L}}{\bar{L}}$, $\hat{L}$ = length of arc AD and $\bar{L}$ = length of chord AD,

(iii) The resultant reaction R passing through the point of intersection of the above two forces and tangential to the friction circle.

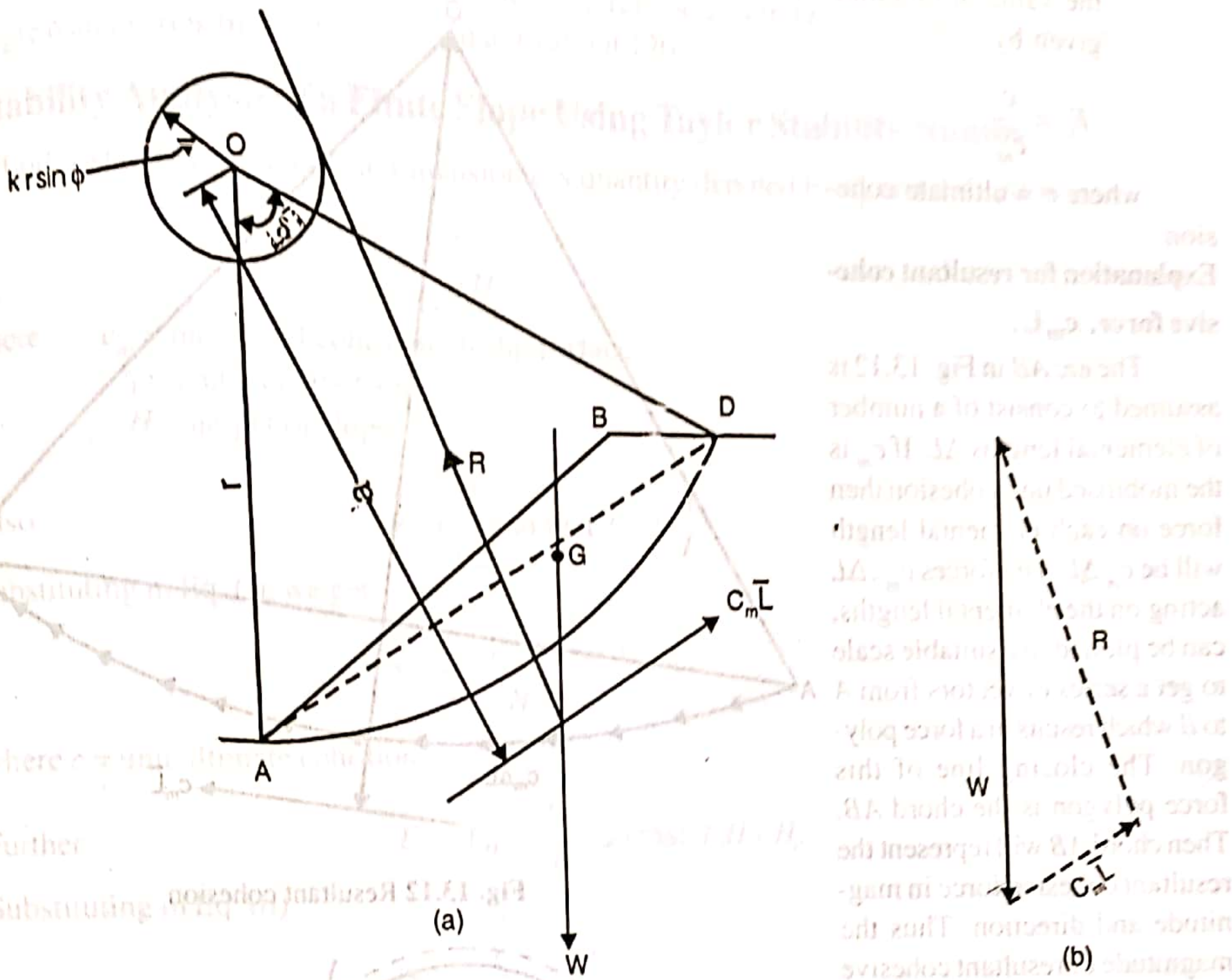

Fig. 13.11 Friction circle method

Procedure :

1. With centre O and radius r, the slip circle AD is constructed. The friction circle is drawn with centre O and radius $Kr \sin \phi$. K is taken as 1 unless otherwise given.
2. A vertical line is drawn through centroid of section $ABDA$, to get the line of action of weight W.
3. Chord AD is drawn. A line is drawn parallel to chord AD and at distance $a = r.\frac{\hat{L}}{\bar{L}}$ from O, to get the line of action of resultant cohesive force $c_m \bar{L}$. The length of arc AD, $\hat{L}$ is computed using the equation $\hat{L} = \frac{\pi r \delta}{180}$. The length of chord AD, $\bar{L}$ is obtained by measurement.
4. Through the point of intersection of the lines of action of forces W and $c_m \bar{L}$, a line is drawn tangential to the friction circle, to get the line of action of resultant reaction R.
5. The weight W of the sliding soil mass $ABDA$ is computed and plotted to scale as shown in Fig. 13.11 (b). Through the ends of the vector representing W, lines are drawn parallel to the lines of action of forces $c_m \bar{L}$ and R to complete the triangle of forces.

6. The value of $c_m \bar{L}$ is obtained from the force triangle and divided by value of $\bar{L}$ to obtain the value of mobilised cohession c_m. The factor of safety with respect to cohesion, F_C is given by

$$F_c = \frac{c}{c_m}$$

where c = ultimate cohesion.

Explanation for resultant cohesive force, $c_m \bar{L}$.

The arc AB in Fig. 13.12 is assumed to consist of a number of elemental lengths ΔL. If c_m is the mobilised unit cohesion then force on each elemental length will be $c_m \Delta L$. The forces $c_m . \Delta L$ acting on the elemental lengths, can be plotted to a suitable scale to get a series of vectors from A to B which results in a force polygon. The closing line of this force polygon is the chord AB. Then chord AB will represent the resultant cohesive force in magnitude and direction. Thus the magnitude of resultant cohesive force is $c_m \bar{L}$ where $\bar{L}$ is the length of chord AB. Let the resultant cohesive force acting parallel to chord AB, be at distance a from O. Taking moments about O and applying Varignon's theorem we get

$$(c_m \bar{L})(a) = \Sigma c_m . \Delta L . r$$

$$= c_m . r . \Sigma \Delta L$$

$$= c_m . r . \hat{L}$$

$$\therefore \quad a = \frac{\hat{L}}{\bar{L}} r$$

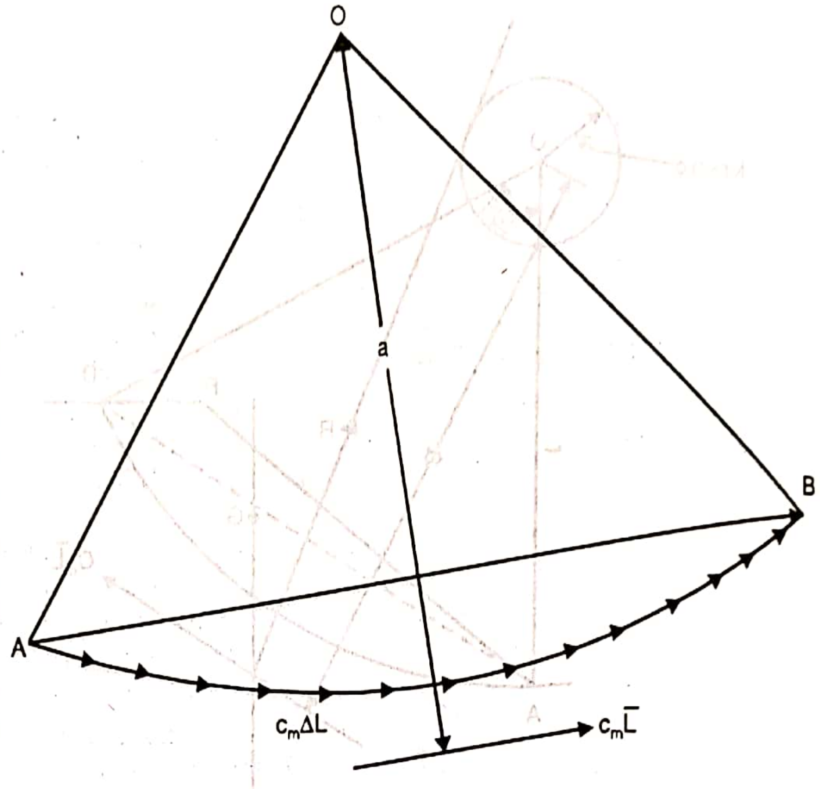

Fig. 13.12 Resultant cohesion

Explanation for resultant reaction R

The arc AB is assumed to consist of a number of elemental lengths ΔL. The resultant reaction ΔR acting on an element ΔL will be inclined at angle ϕ to the normal at the mid-point of

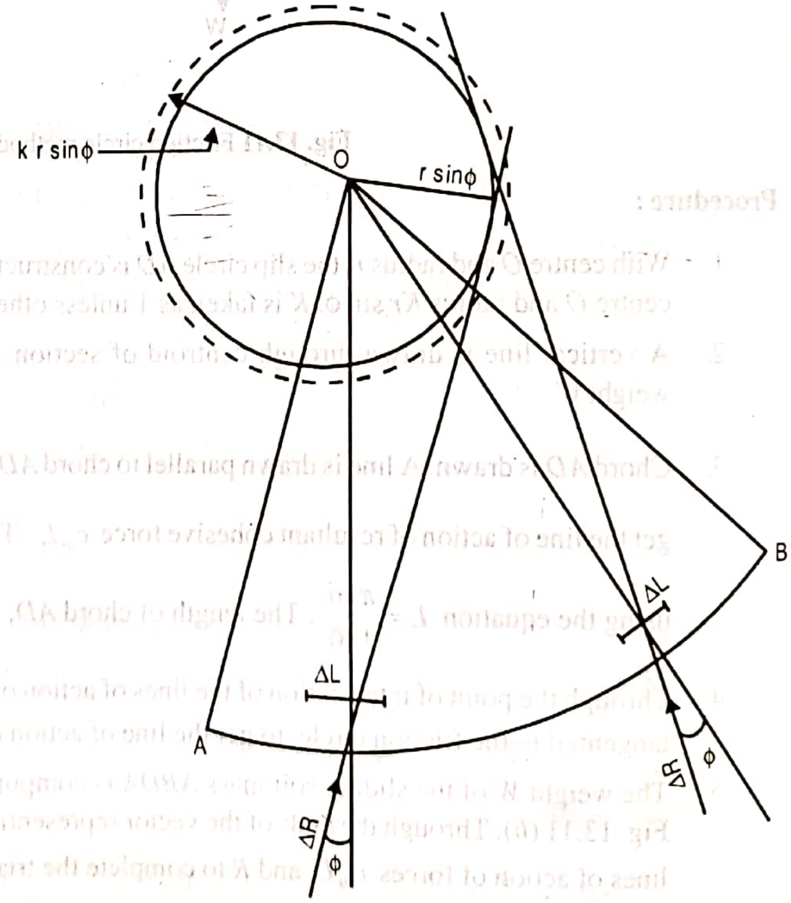

Fig. 13.13 Resultant reaction R.

the element, so that it will be tangential to the circle of radius $r \sin \phi$ drawn with centre O. It can be shown that the resultant reaction R for the entire arc AB will be tangential to the circle of radius $Kr \sin \phi$, drawn with centre O. This circle is called friction circle or ϕ - circle. The value of K depends on central angle δ and varies from 1 to 1.12 for δ upto about 120°.

13.8 Stability Analysis of a Finite Slope Using Taylor Stability Number

Taylor stability number is a dimensionless quantity denoted by S_n and defined as

$$S_n = \frac{c_m}{\gamma H} \quad ...(i)$$

where c_m = mobilised cohesion on slip surface

γ = unit weight of soil

H = height of slope

Also $F_c = \frac{c}{c_m}$ so that $c_m = \frac{c}{F_c}$

Substituting in Eq. (i), we get

$$S_n = \frac{c}{F_c \gamma H} \quad ...(ii)$$

where c = unit ultimate cohesion.

Further $F_c = F_H = \frac{H_c}{H}$ so that $F_c H = H_c$

Substituting in Eq. (ii)

$$S_n = \frac{c}{\gamma H_c} \quad ...(iii)$$

where H_c = critical height of slope.

Using the friction circle method along with an analytical procedure, Taylor determined S_n for finite slopes and presented the results in the form of a table (Table 13.1) and a chart (Fig. 13.19) from which one can obtain value of S_n for different values of slope angle i and angle of shearing resistance ϕ.

Since Taylor stability number S_n is based on factor of safety with respect to cohesion, F_c, the table and chart give S_n only for the case where ϕ is assumed to be fully mobilised. But in cases where factor of safety is applicable to both cohesion and friction, we have mobilised shearing resistance given by

$$\tau_m = \frac{\tau_f}{F} = \frac{c}{F} + \frac{\sigma \tan \phi}{F}$$

While obtaining S_n from chart, mobilised angle of shearing resistance ϕ_m should be used.

We have $\tan \phi_m = \frac{\tan \phi}{F}$

$$\therefore \quad \phi_m = \tan^{-1}\left(\frac{\tan \phi}{F}\right)$$

As an approximation ϕ_m may be taken equal to $\frac{\phi}{F}$.

For a cohesionless soil ($c = 0$) the Taylor stability number is zero and Taylor's chart is not applicable. The factor of safety is given by $F = \frac{\tan \phi}{\tan i}$ and is independent of height of slope.

For long term stability c' and ϕ' obtained from drained test should be used. Use of Taylor's stability number gives an approximate idea of long term stability, if seepage effect can be neglected and no change in water content can be assumed.

In the case of fully submerged slopes, γ' should be used in the expression for S_n. When the slope is saturated, as for example by capillary water, γ_{sat} should be used in the expression for S_n. In the case of sudden drawdown γ_{sat} should be used in the expression for S_n and value of S_n should be obtained from Taylor's chart, corresponding to weighted frictional angle ϕ_w given by $\phi_w = \frac{\gamma'}{\gamma_{sat}}\phi$

Taylor also determined stability number S_n for different values of slope angle i and depth factor D_f. The depth factor D_f is defined as the ratio of depth to hard strata below top of slope to the height of slope. This is illustrated in Fig. 13.14.

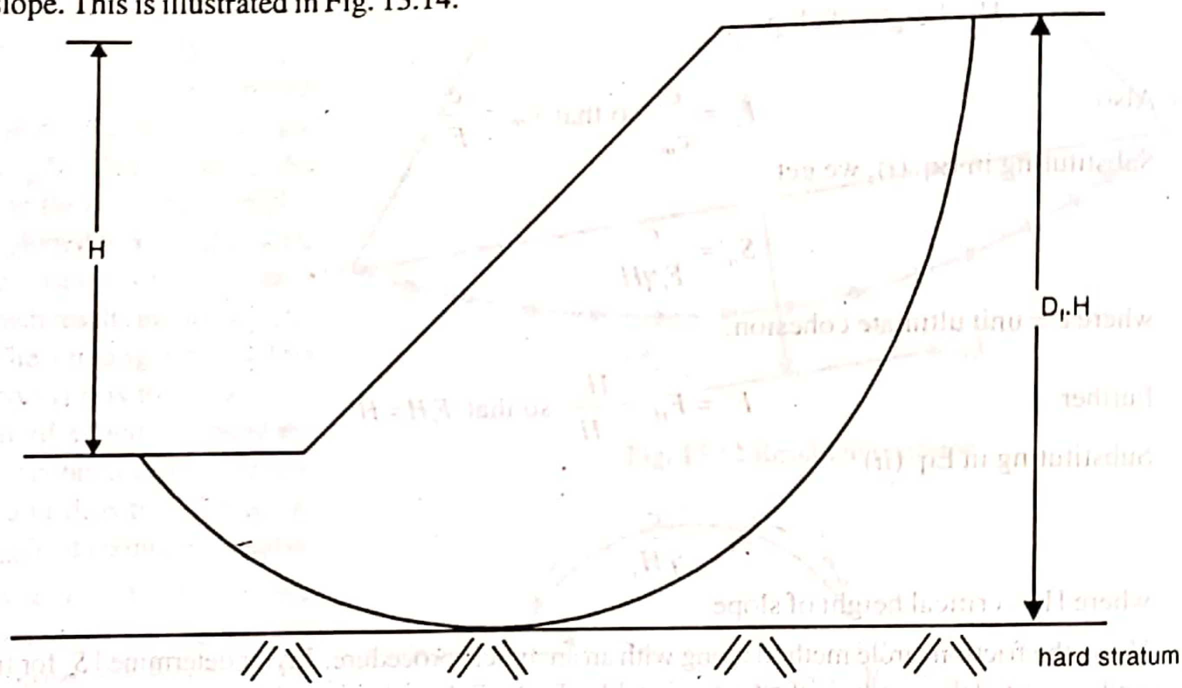

Fig. 13.14 Illustration of depth factor, D_f

Taylor presented table (Table 13.2) and chart (Fig. 13.19) from which one can obtain S_n for different values of i and D_f.

13.9 Bishop's Method of Stability Analysis

In the Bishop's method of stability analysis the following assumptions are made :

1. The slip surface is cylindrical, that is, arc of a circle in section.
2. The sliding soil mass is assumed to consist of a number of vertical slices.
3. The forces of interaction between adjacent slices, which were neglected in the Swedish method, are considered in the Bishop's method.

Analysis. Let AD be a slip circle with radius r and O the centre of rotation. The section of sliding soil mass $ABDA$ is divided into a number of slices. In the Fig. 13.15 is shown the free body diagram of a slice between sections n and $n+1$. The explanation of notations is as follows.

E_n and E_{n+1} = normal forces on the sections n and $n+1$, exerted by adjacent slices.

X_n and X_{n+1} = shear forces on sections n and $n+1$, exerted by adjacent slices.

W = weight of slice

N = normal reaction at the base of slice