Geotechnical Engineering–II [CE-321]

BSc Civil Engineering – 5th Semester

Lecture # 23

29-Nov-2017

by

Dr. Muhammad Irfan

Assistant Professor

Civil Engg. Dept. – UET Lahore

Email: mirfan1@msn.com

Lecture Handouts: https://groups.google.com/d/forum/geotech-ii_2015session

RANKINE THEORY OF ACTIVE EARTH PRESSURE

ASSUMPTIONS

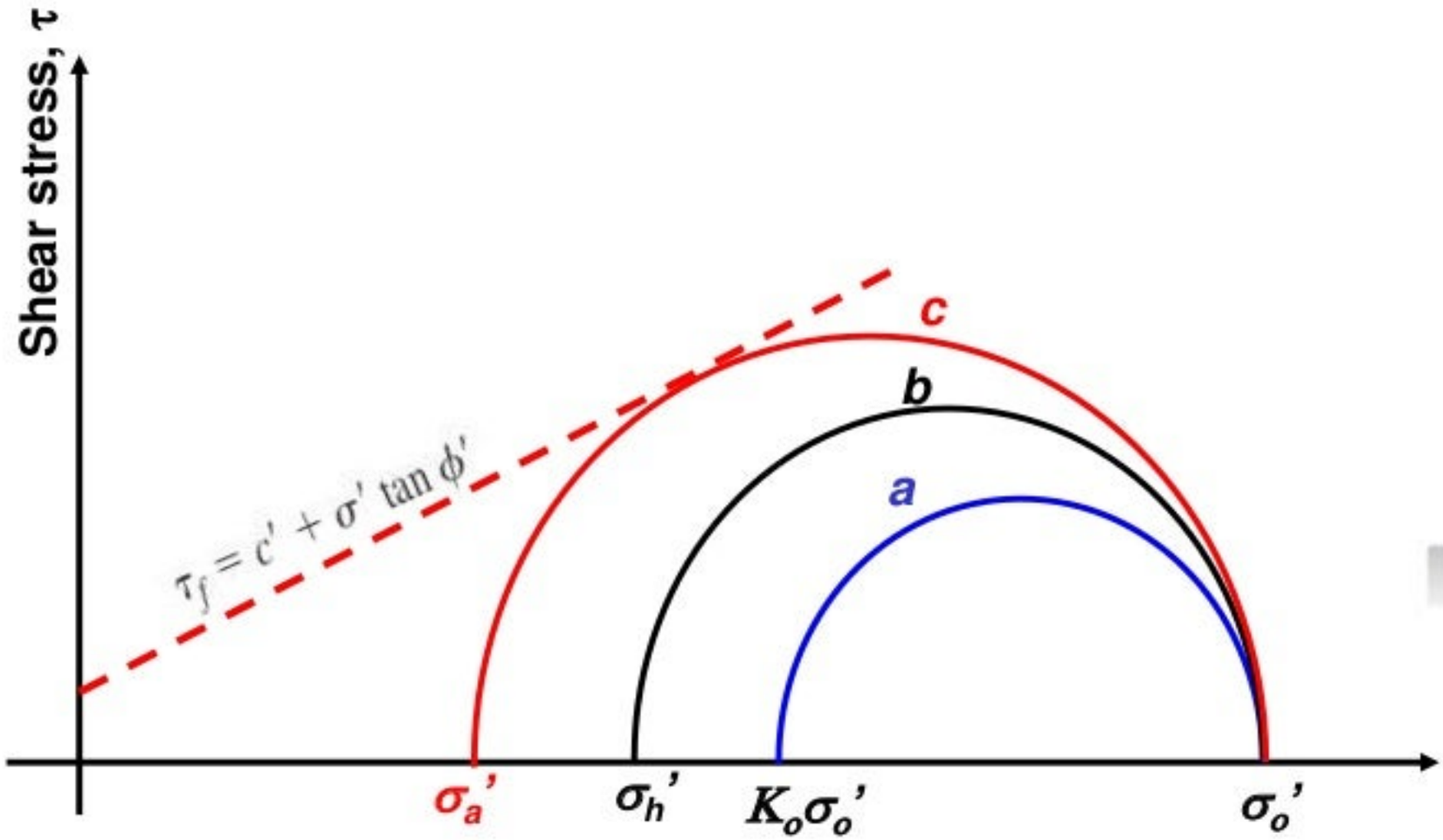
1. The soil is *homogeneous* and *isotropic*.
2. The most *critical shear surface is a plane*. *In reality, it is slightly concave upward, but this is a reasonable assumption (especially for the active case) and it simplifies the analysis.*
3. The *backfill surface is planar* *(although it does not necessarily need to be horizontal)*.
4. There is *no friction* between wall and soil.
5. The *wall is infinitely long* so that a representative two-dimensional section of the wall may be analyzed, assuming there is no strain in the direction perpendicular to the section. We refer to this as a *plane strain condition*.

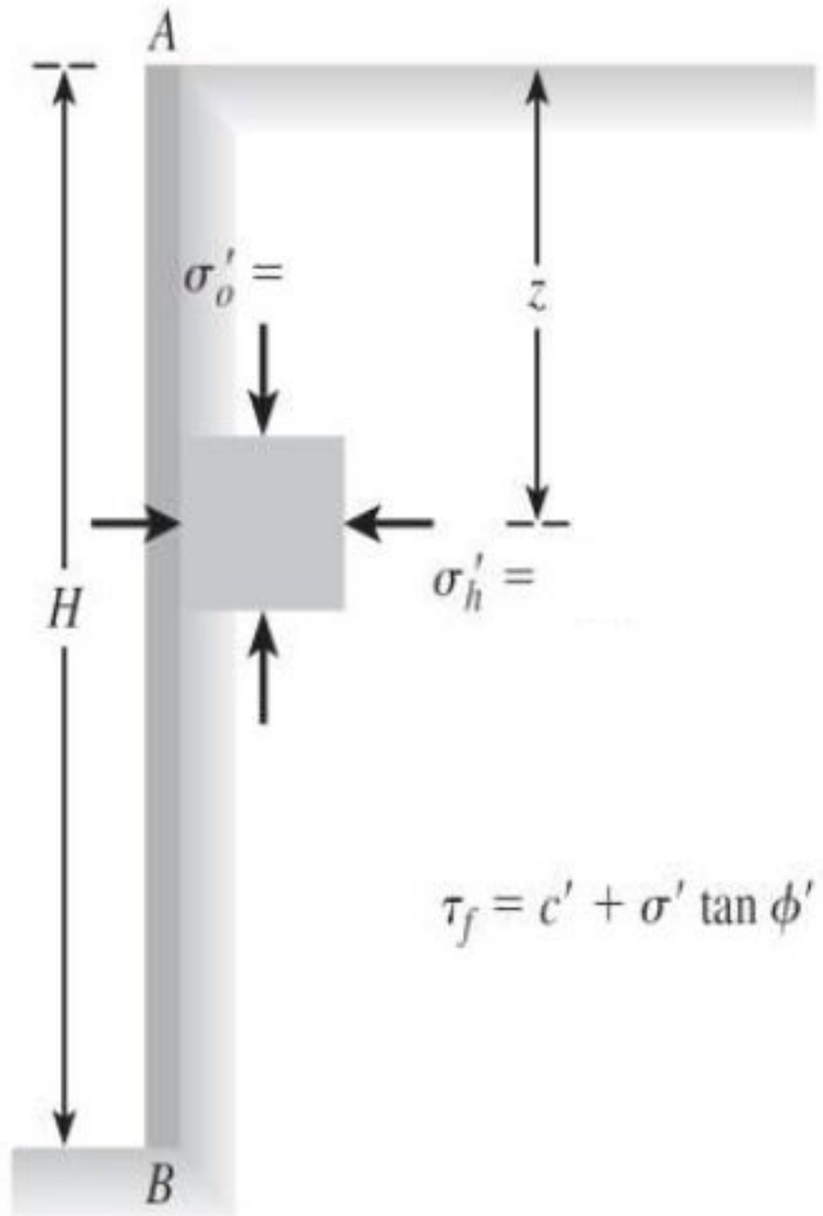
RANKINE THEORY OF ACTIVE EARTH PRESSURE

If wall AB is not allowed to move → $\sigma'_h = K_o \sigma'_o$

Stress condition in soil → Mohr's circle *a*

If wall is allowed to move away from soil mass
→ horizontal principal stress will decrease →
→ Circle *b* → Circle *c*

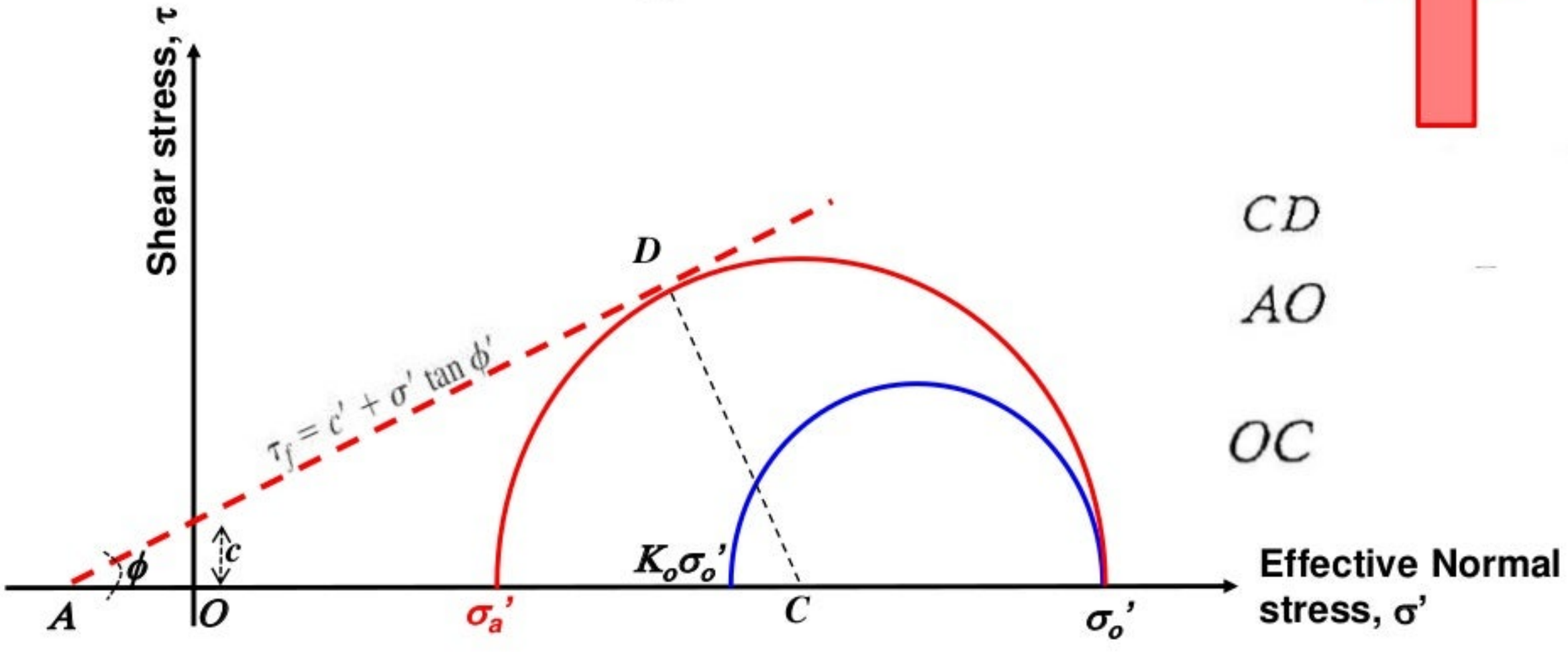




RANKINE THEORY OF ACTIVE EARTH PRESSURE

$$\sin\phi' = \frac{CD}{AC} = \frac{CD}{AO + OC}$$

$$\sin\phi' = \frac{\dfrac{\sigma'_o - \sigma'_a}{2}}{c' \cot\phi' + \dfrac{\sigma'_o + \sigma'_a}{2}}$$



RANKINE THEORY OF ACTIVE EARTH PRESSURE

$$\sin\phi' = \frac{\dfrac{\sigma'_o - \sigma'_a}{2}}{c'\cot\phi' + \dfrac{\sigma'_o + \sigma'_a}{2}}$$

$$c'\cos\phi' + \frac{\sigma'_o + \sigma'_a}{2}\sin\phi' = \frac{\sigma'_o - \sigma'_a}{2}$$

$$\sigma'_a = \sigma'_o\frac{1 - \sin\phi'}{1 + \sin\phi'} - 2c'\frac{\cos\phi'}{1 + \sin\phi'}$$

RANKINE THEORY OF ACTIVE EARTH PRESSURE

$$\sigma'_a = \sigma'_o \frac{1 - \sin\phi'}{1 + \sin\phi'} - 2c' \frac{\cos\phi'}{1 + \sin\phi'}$$

σ'_o = vertical effective overburden pressure = γz

$$\frac{1 - \sin\phi'}{1 + \sin\phi'} = \tan^2\left(45 - \frac{\phi'}{2}\right)$$

$$\frac{\cos\phi'}{1 + \sin\phi'} = \tan\left(45 - \frac{\phi'}{2}\right)$$

$$\sigma'_a = \gamma z \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \tan\left(45 - \frac{\phi'}{2}\right)$$

RANKINE THEORY OF ACTIVE EARTH PRESSURE

$$\sigma'_a = \gamma z \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \tan\left(45 - \frac{\phi'}{2}\right)$$

For cohesionless soils, c' = 0

$$\sigma'_a = \sigma'_o \tan^2\left(45 - \frac{\phi'}{2}\right)$$

$$K_a = \frac{\sigma'_a}{\sigma'_o} = \tan^2\left(45 - \frac{\phi'}{2}\right)$$

RANKINE THEORY OF ACTIVE EARTH PRESSURE

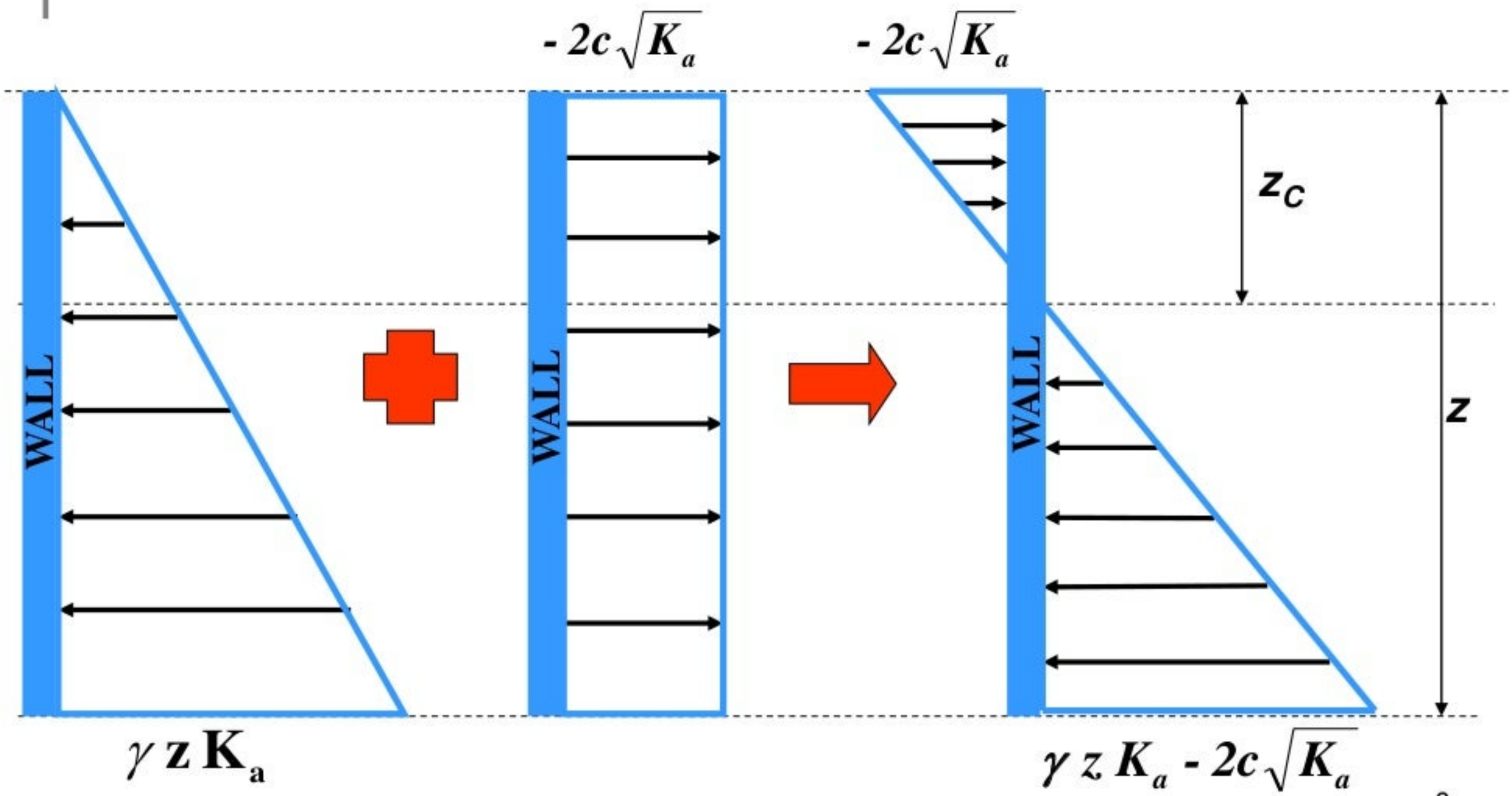
$$\sigma'_a = \gamma z \cdot \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \cdot \tan\left(45 - \frac{\phi'}{2}\right)$$

$$K_a = \frac{\sigma'_a}{\sigma'_o} = \tan^2\left(45 - \frac{\phi'}{2}\right) = \frac{1 - \sin\phi'}{1 + \sin\phi'}$$

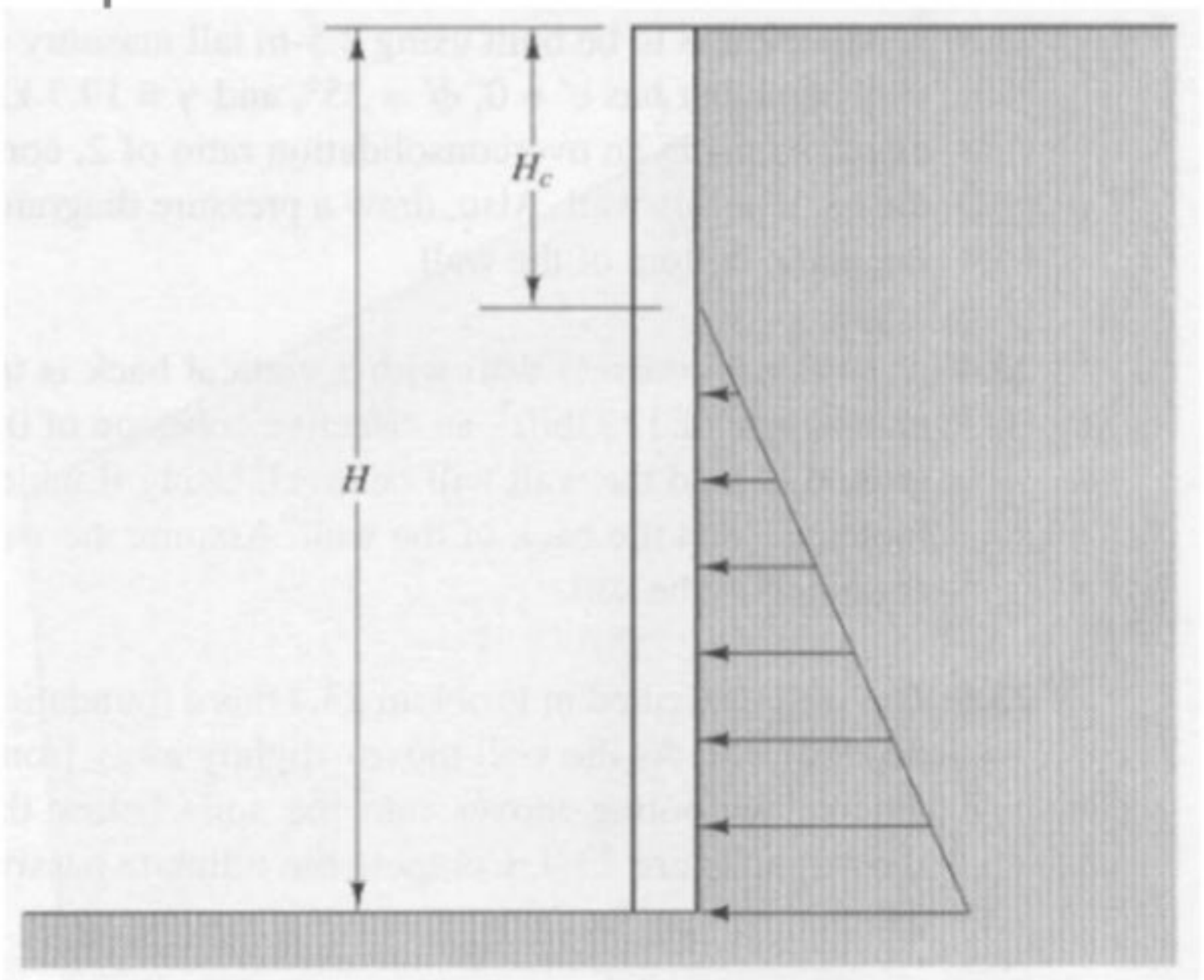
$$\sigma'_a = \gamma z K_a - 2c'\sqrt{K_a}$$

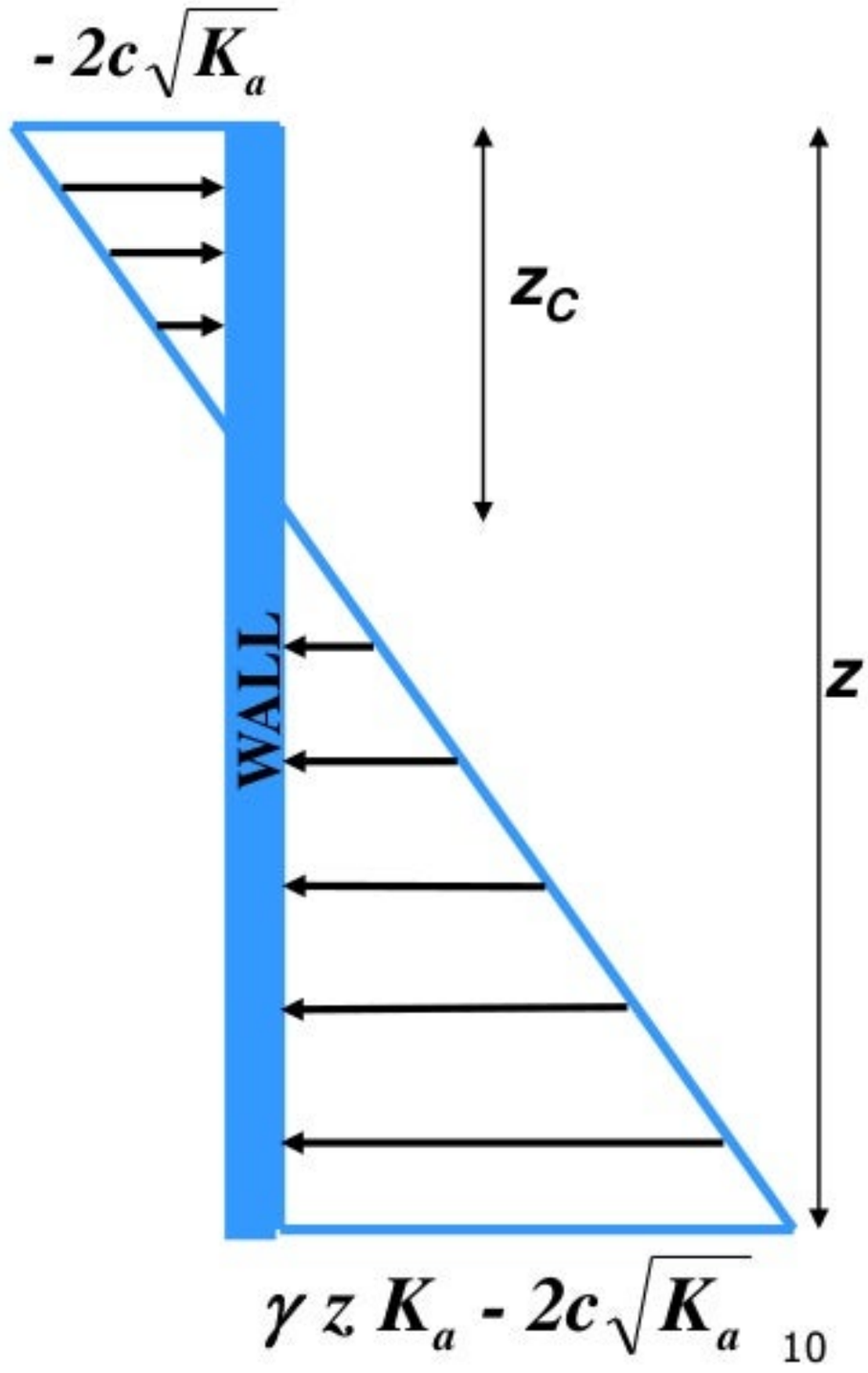
RANKINE ACTIVE PRESSURE DIAGRAM

$$\sigma'_a = \gamma z K_a - 2c'\sqrt{K_a}$$



RANKINE ACTIVE PRESSURE DIAGRAM





RANKINE THEORY OF ACTIVE EARTH PRESSURE
-- DEPTH OF UNSUPPORTED CUT --

$$\sigma'_a = \gamma z K_a - 2c'\sqrt{K_a}$$

General Case (c-ϕ soil)

At $z_C \rightarrow \sigma'_a = 0$

$$0 = \gamma z_c K_a - 2c'\sqrt{K_a}$$

$$z_c = \frac{2c'\sqrt{K_a}}{\gamma K_a}$$

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}}$$

Cohesionless Soils (c'=0)

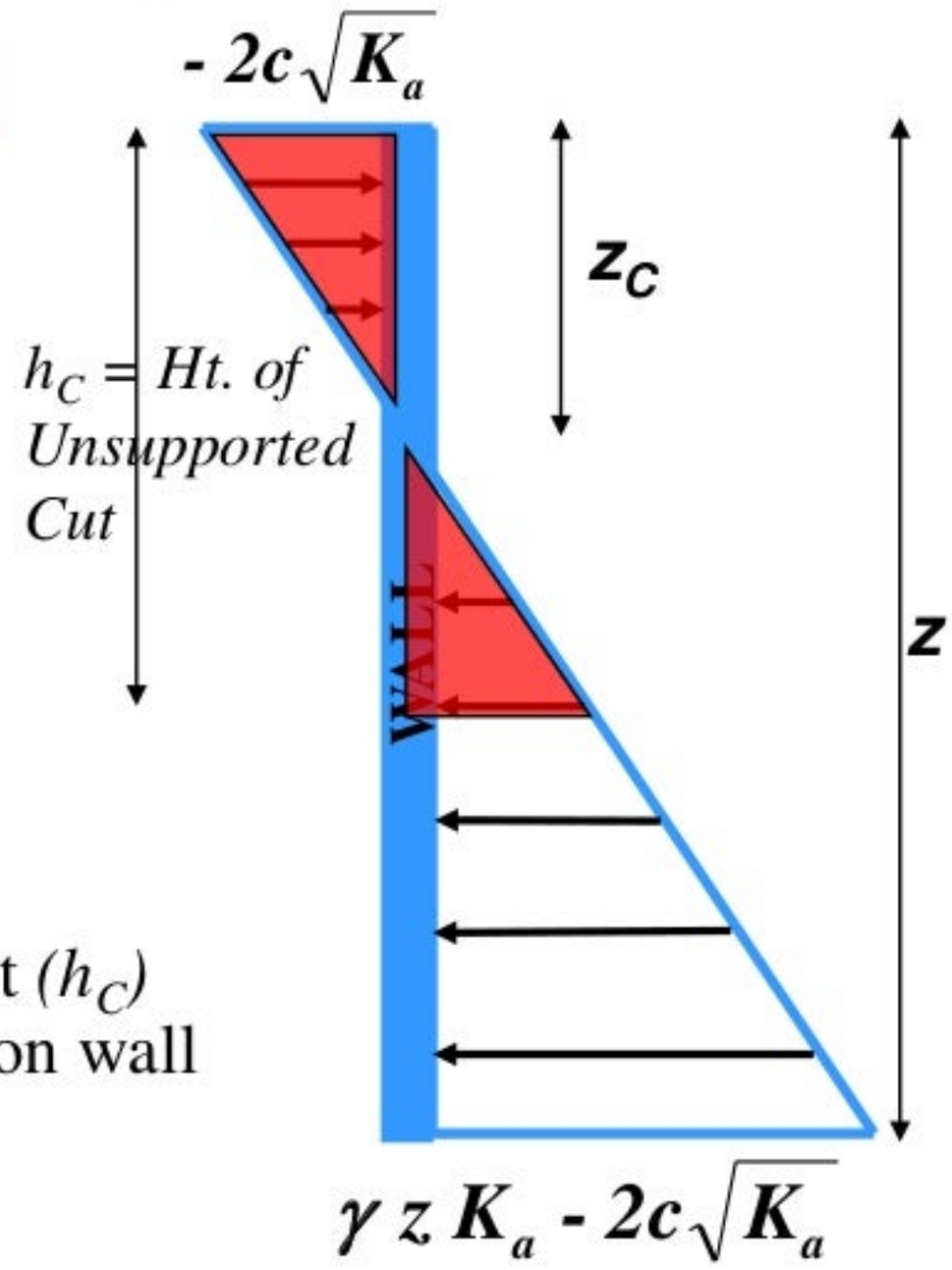
$$z_C = 0$$

Cohesive Soils (ϕ=0)

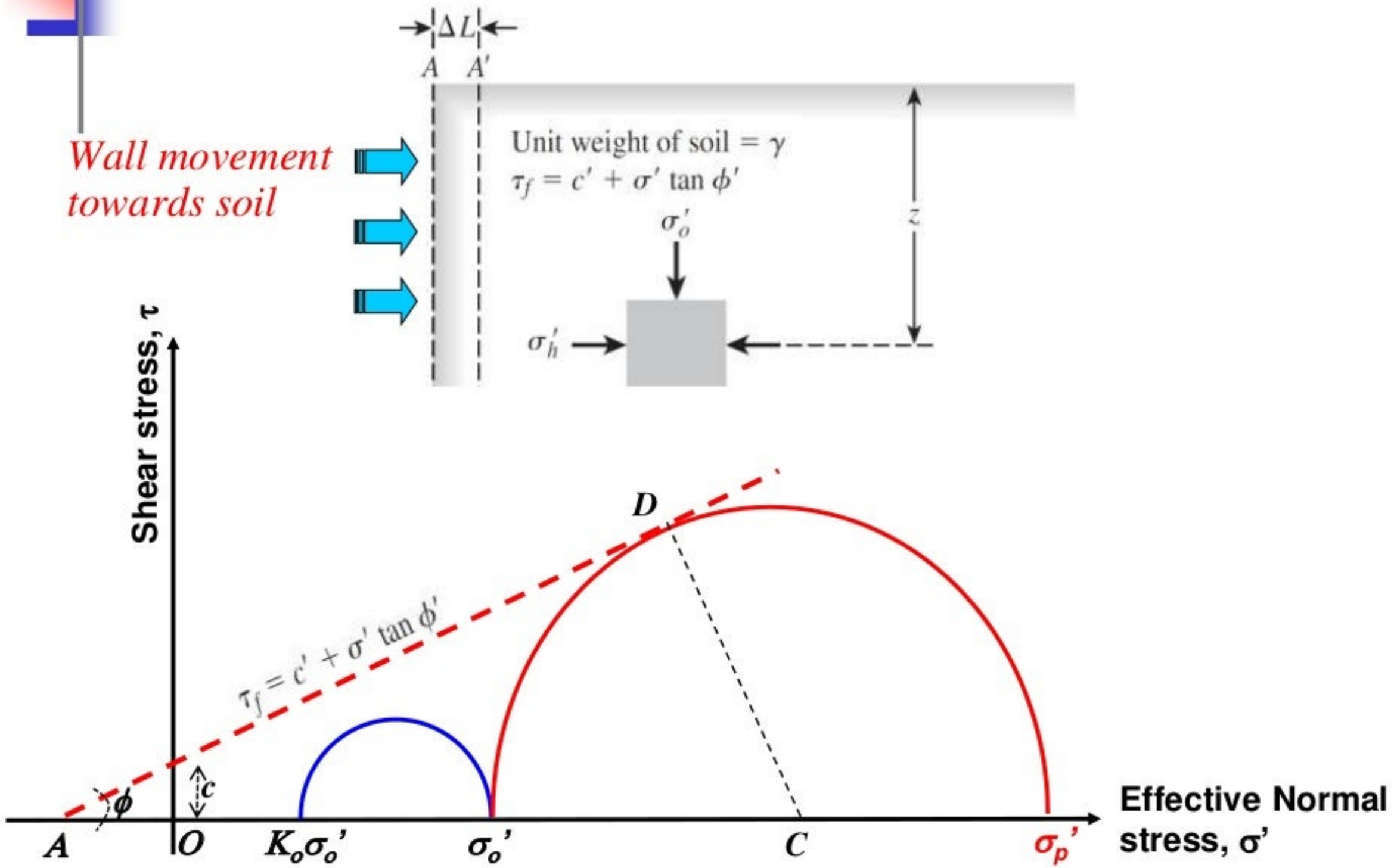
$K_a = 1 \Rightarrow z_c = \frac{2c'}{\gamma}$

Height of unsupported cut (h_C)
→ Zero Net force acting on wall

$$h_C = 2 \times z_C$$



RANKINE THEORY OF PASSIVE EARTH PRESSURE



RANKINE THEORY OF PASSIVE EARTH PRESSURE

Derivation: similar to that for active state

$$\sigma'_p = \sigma'_o \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

$$= \gamma z \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

For cohesionless soils, c' = 0

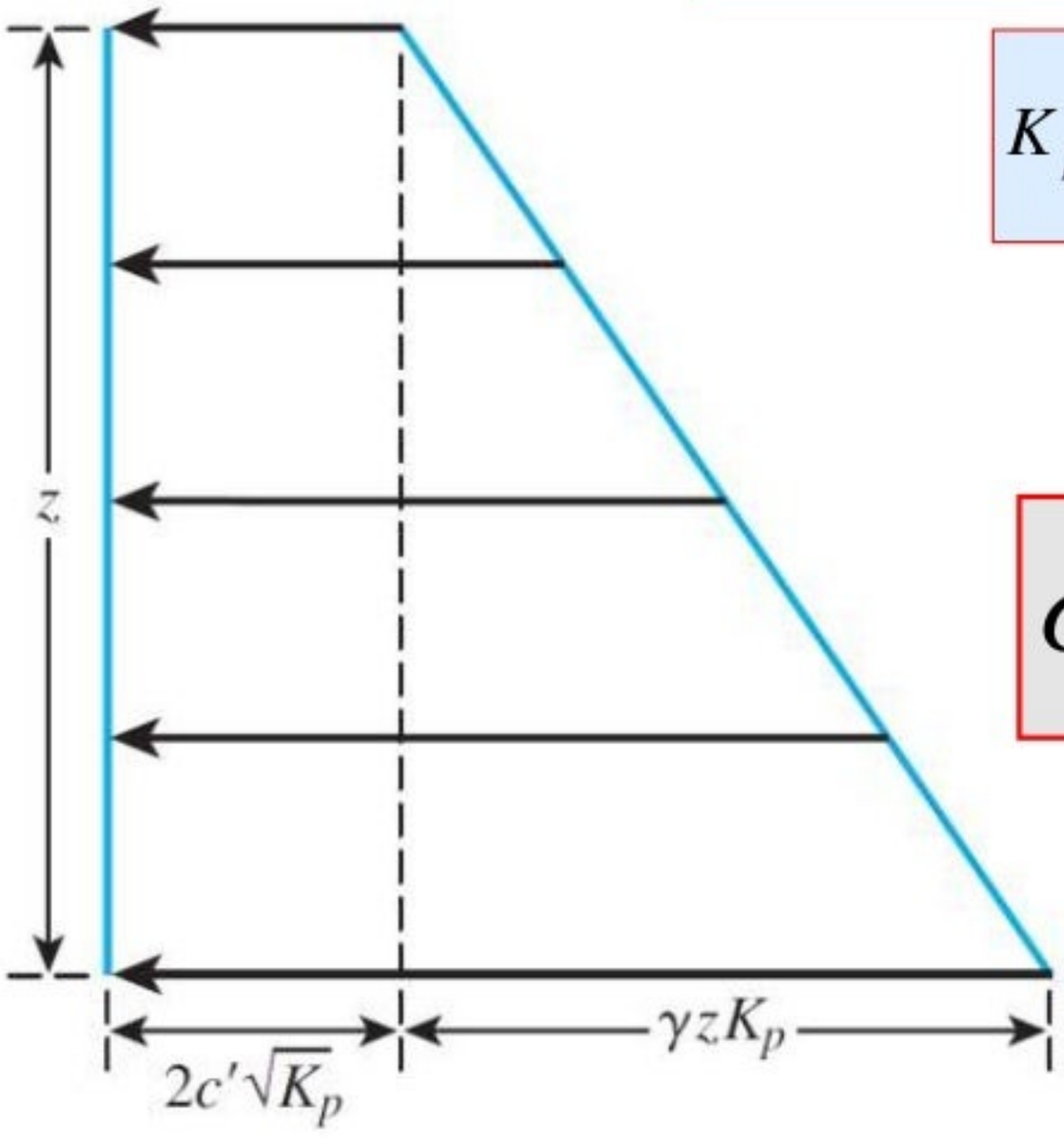
$$\sigma'_p = \sigma'_o \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{\sigma'_p}{\sigma'_o} = K_p = \tan^2\left(45 + \frac{\phi'}{2}\right)$$

RANKINE THEORY OF PASSIVE EARTH PRESSURE

$$\sigma'_p = \gamma z \cdot \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \cdot \tan\left(45 + \frac{\phi'}{2}\right)$$

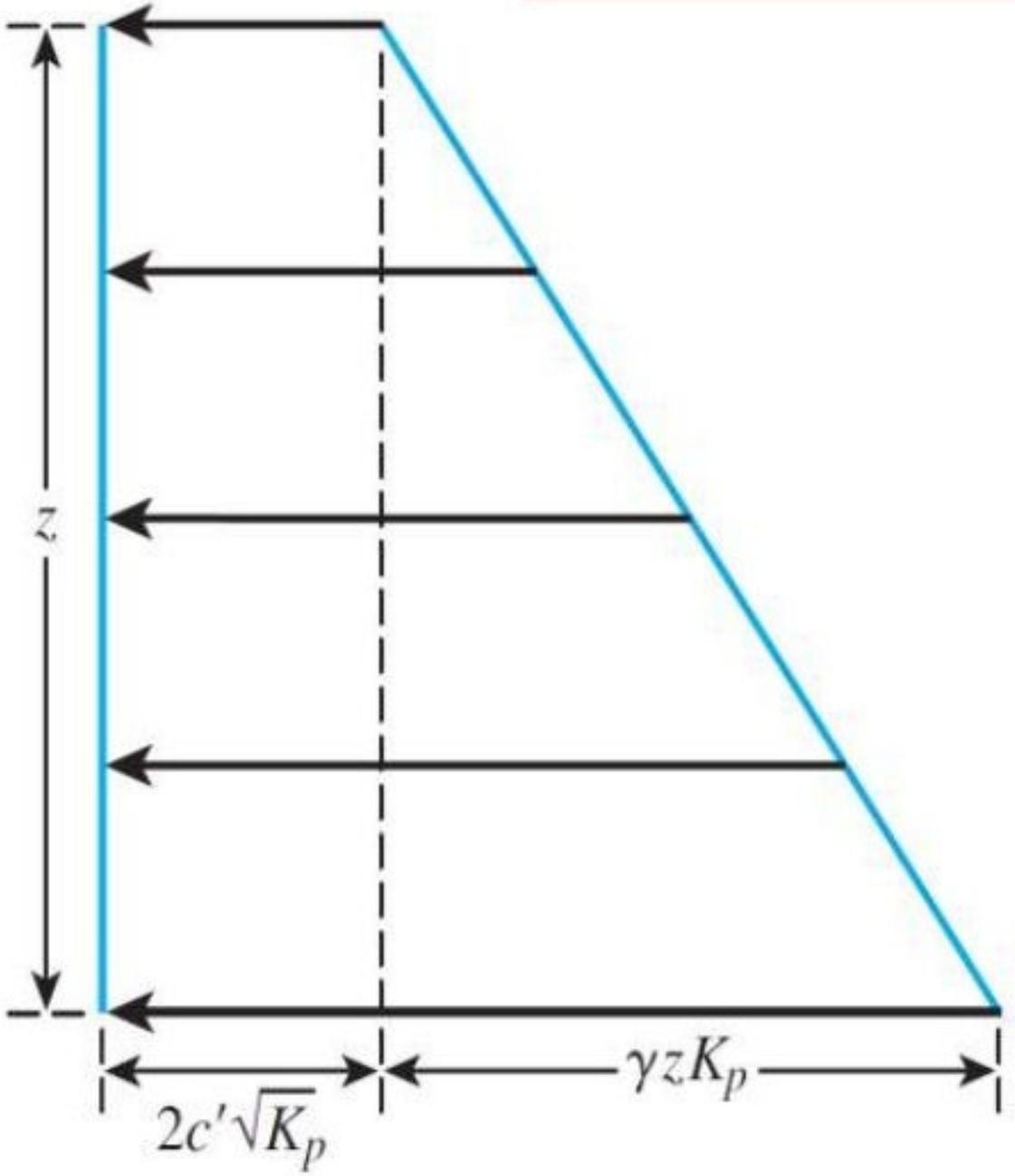
$$K_p = \frac{\sigma'_p}{\sigma'_o} = \tan^2\left(45 + \frac{\phi'}{2}\right) = \frac{1 + \sin\phi'}{1 - \sin\phi'}$$

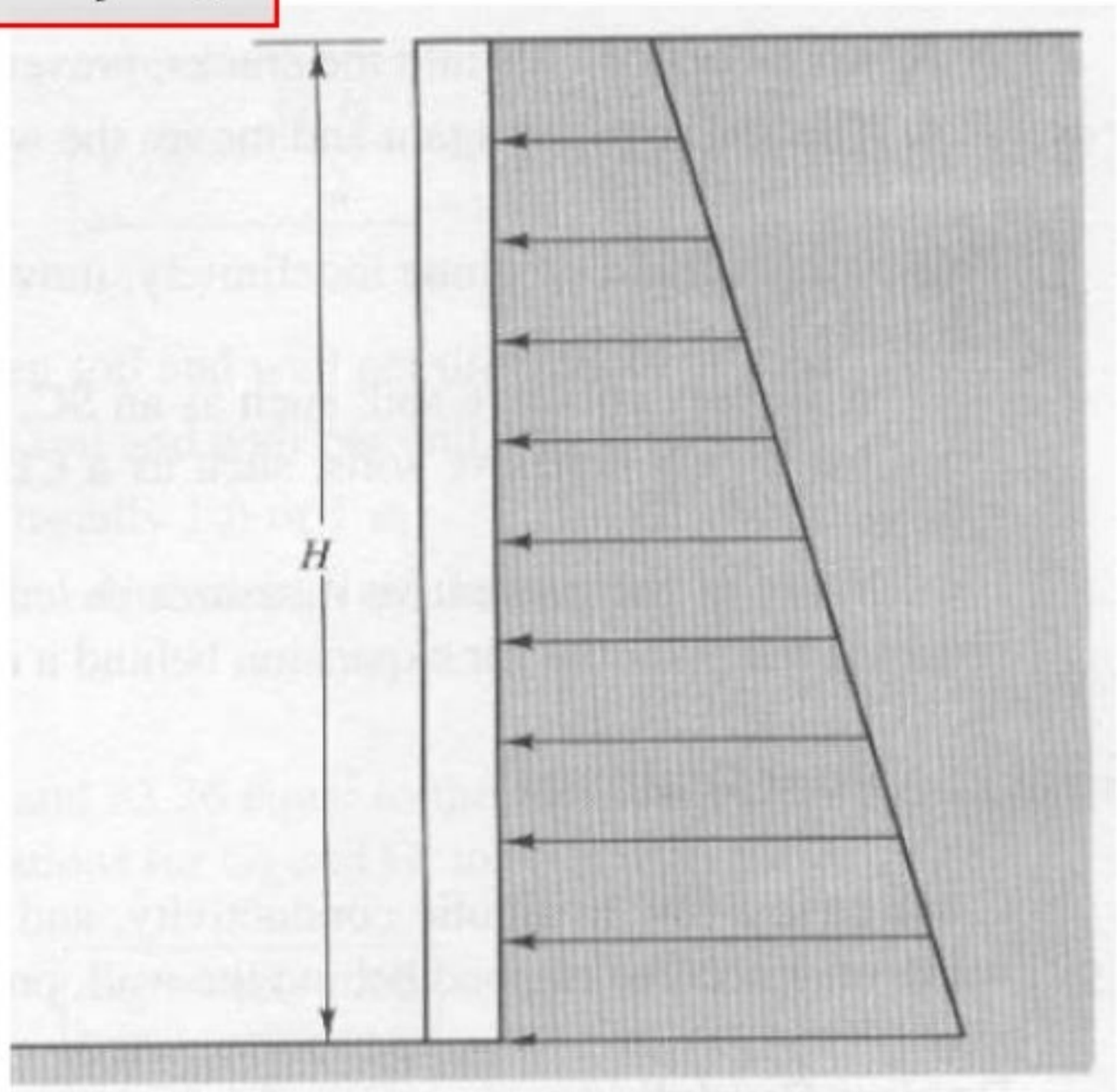


$$\sigma'_p = \gamma z K_p + 2c'\sqrt{K_p}$$

RANKINE THEORY OF PASSIVE EARTH PRESSURE

$$\sigma'_p = \gamma z K_p + 2c'\sqrt{K_p}$$

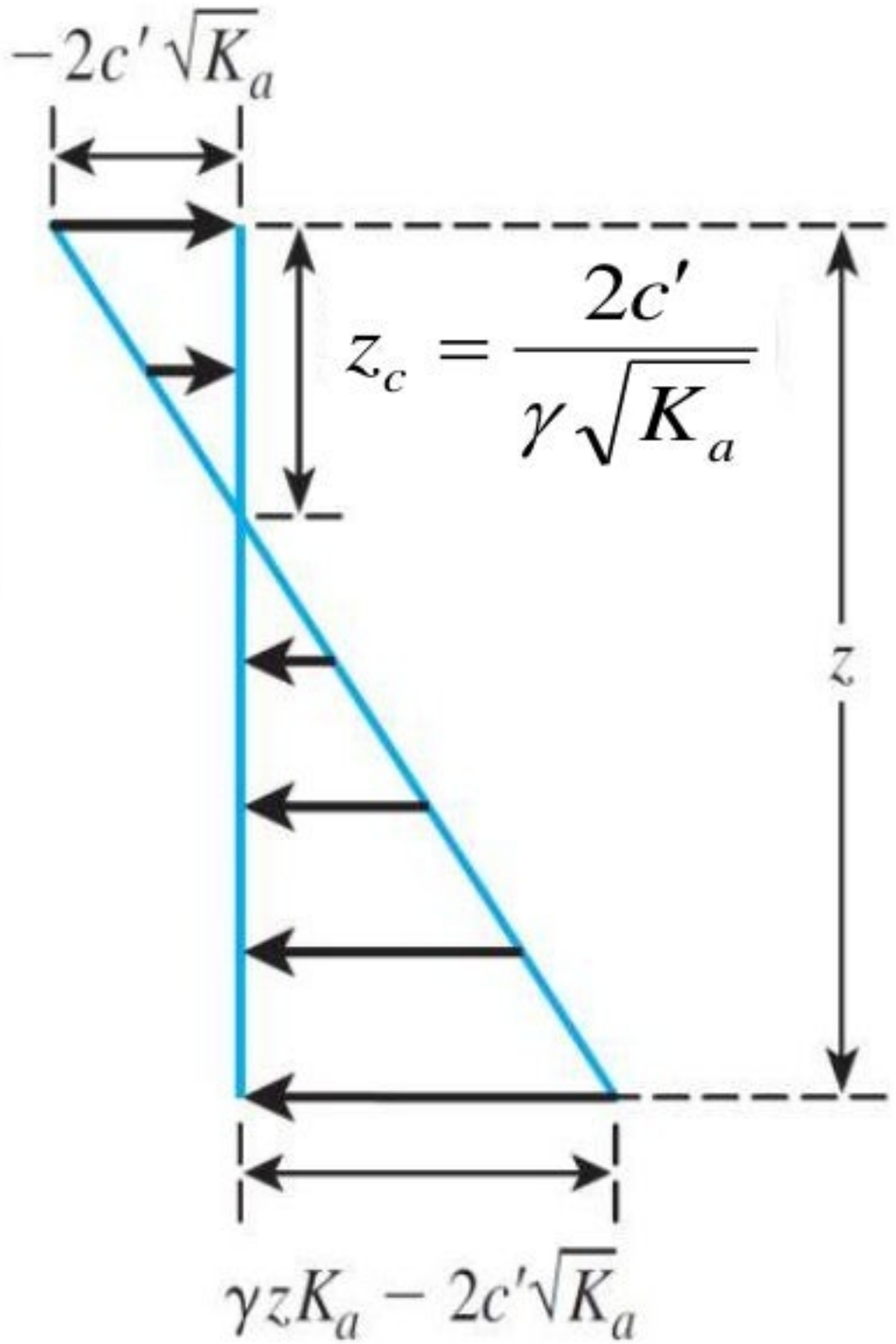




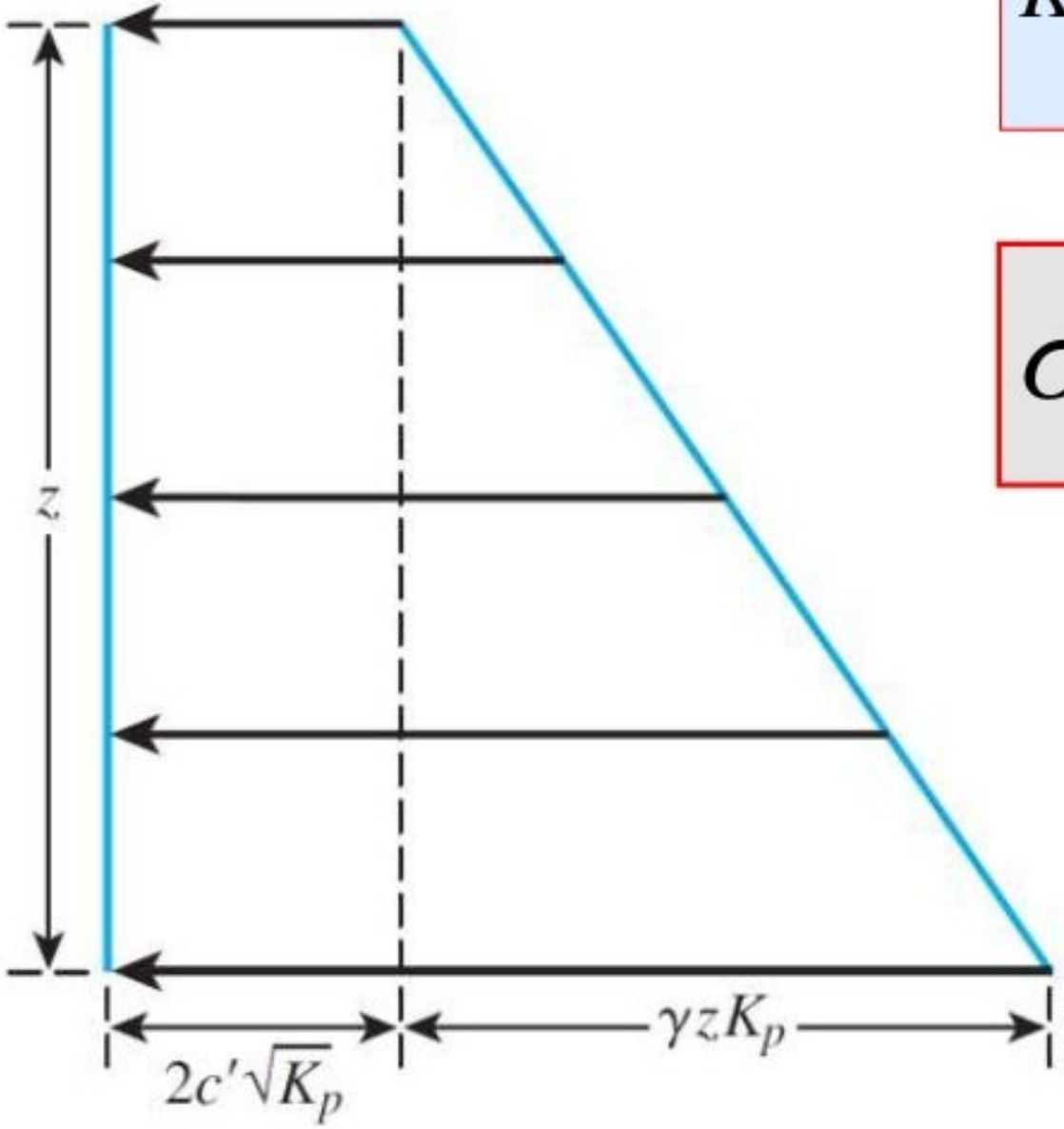
RANKINE THEORY
ACTIVE PRESSURE -- SUMMARY --

$$K_a = \frac{\sigma'_a}{\sigma'_o} = \tan^2\left(45 - \frac{\phi'}{2}\right) = \frac{1-\sin\phi'}{1+\sin\phi'}$$

$$\sigma'_a = \gamma z K_a - 2c'\sqrt{K_a}$$



RANKINE THEORY
PASSIVE PRESSURE -- SUMMARY --

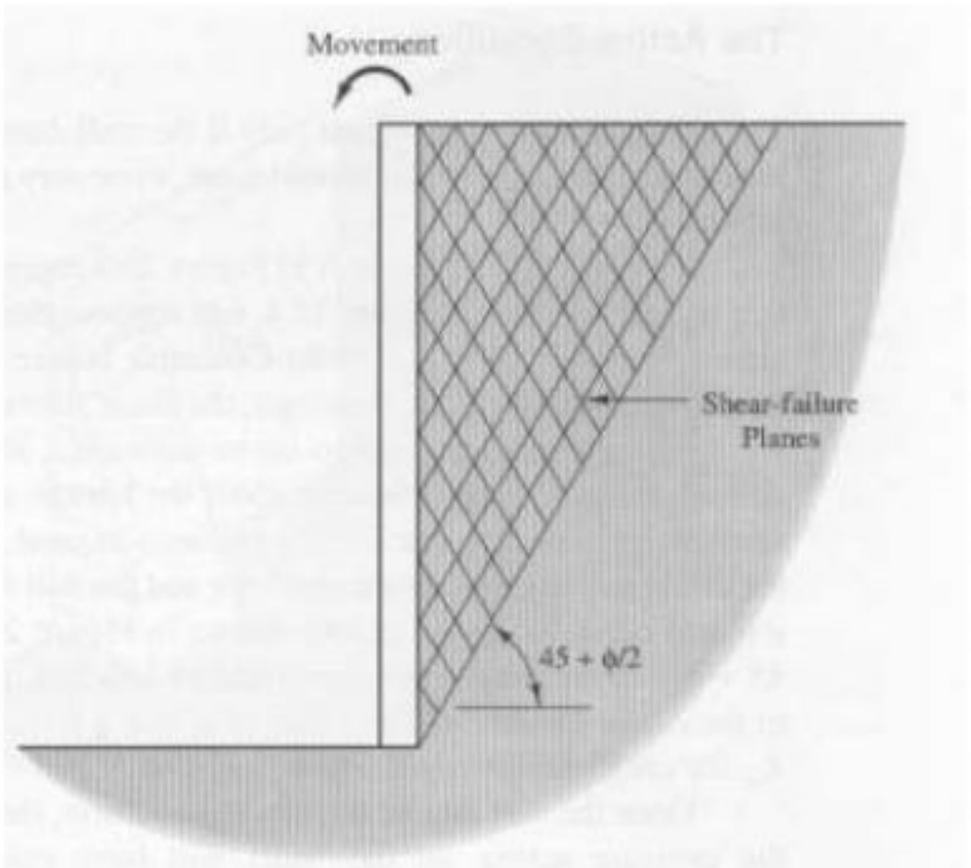


$$K_p = \frac{\sigma'_p}{\sigma'_o} = \tan^2\left(45 + \frac{\phi'}{2}\right) = \frac{1+\sin\phi'}{1-\sin\phi'}$$

$$\sigma'_p = \gamma z K_p + 2c'\sqrt{K_p}$$

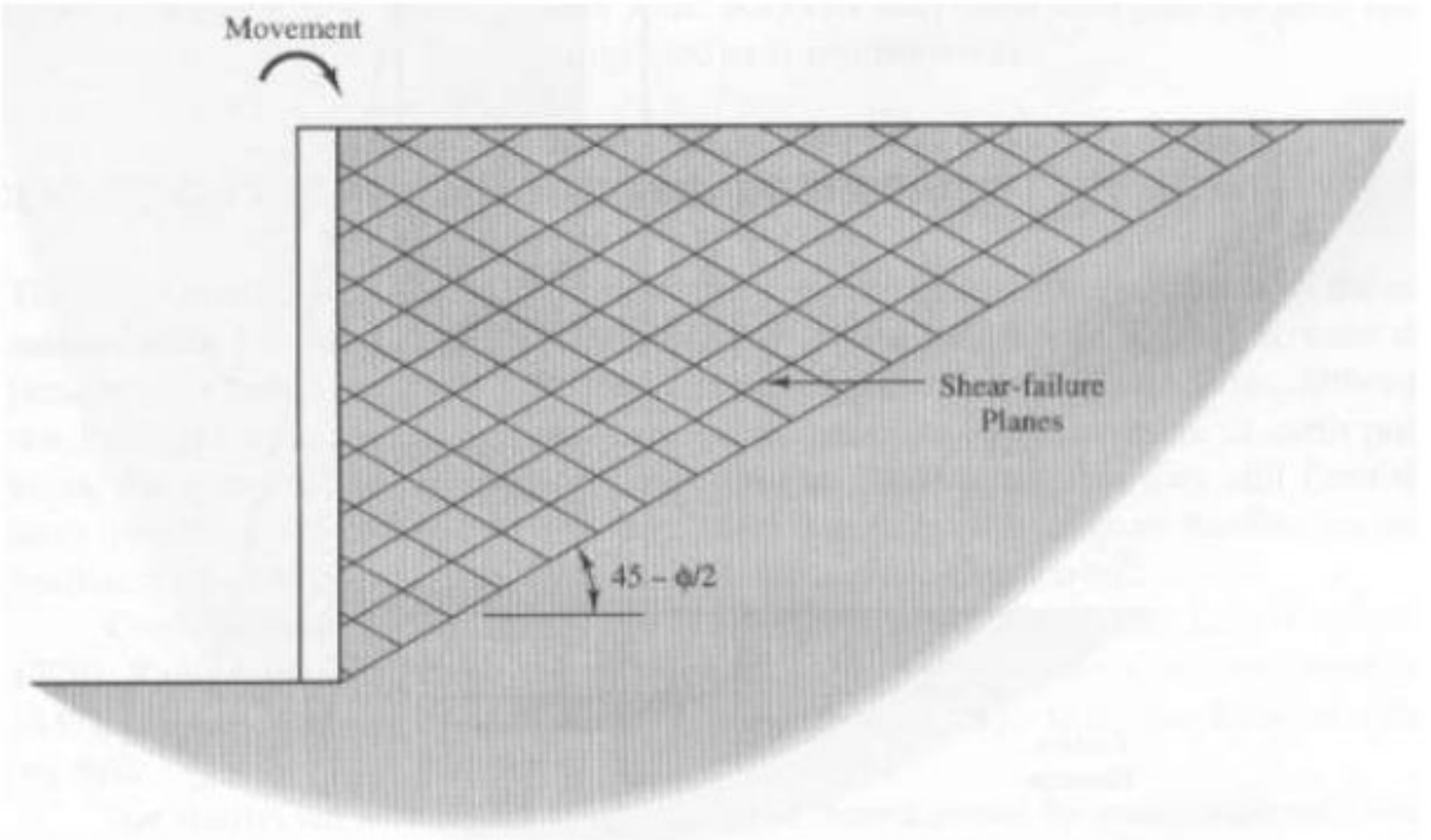
RANKINE THEORY
-- INCLINATION OF FAILURE PLANE --

Active Case



$$Failure\ plane\ inclination = \tan\left(45 + \frac{\phi'}{2}\right)$$

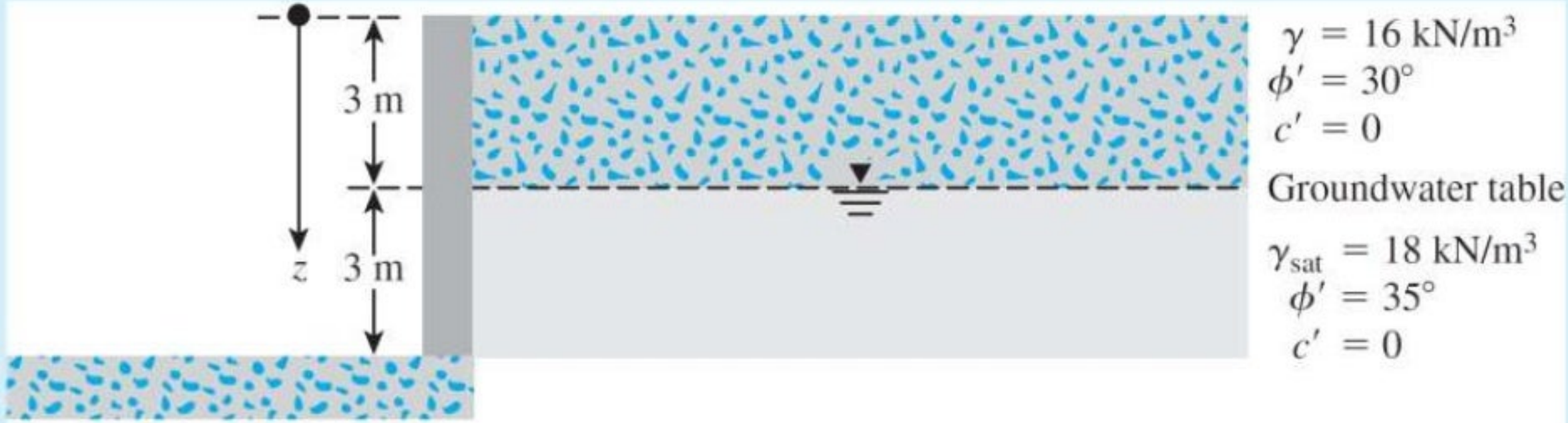
Passive Case



$$Failure\ plane\ inclination = \tan\left(45 - \frac{\phi'}{2}\right)$$

Practice Problem #3

For the retaining wall shown in Figure 13.19a, determine the force per unit length of the wall for Rankine's active state. Also find the location of the resultant.



$$K_a = \frac{\sigma'_a}{\sigma'_o} = \tan^2\left(45 - \frac{\phi'}{2}\right) = \frac{1 - \sin\phi'}{1 + \sin\phi'}$$

$$\sigma'_a = \gamma z K_a - 2c'\sqrt{K_a}$$

REFERENCE MATERIAL

Principles of Geotechnical Engineering – (7th Edition)
Braja M. Das
Chapter #13

Essentials of Soil Mechanics and Foundations (7th Edition)
David F. McCarthy
Chapter #17

Geotechnical Engineering – Principles and Practices – (2nd Edition)
Coduto, Yueng, and Kitch
Chapter #17

CONCLUDED