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LIST OF FORMULAE

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(x + a)(x + b) = x^2 + x(a + b) + ab$
4. $(x + a)(x + b)(x + c) = x^3 + x^2(a + b + c) + x(ab + bc + ca) + abc$
5. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
6. $(a + b)^3 = a^3 + 3ab(a + b) + b^3$
7. $(a - b)^3 = a^3 - 3ab(a - b) - b^3$
8. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
9. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
10. $(a^2 - b^2) = (a + b)(a - b)$
11. $(a^2 + b^2) = (a + ib)(a - ib)$
12. $(a^4 + a^2b^2 + b^4) = (a^2 - ab + b^2)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - 2ab - 2bc - 2ca)$
14. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
15. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
16. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
17. $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$
18. $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$
19. $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$
20. $\sin 3A = 3 \sin A - 4 \sin^3 A$
21. $\cos 3A = 4 \cos^3 A - 3 \cos A$
22. $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$
23. $\sin C + \sin D = 2 \sin \left(\frac{C + D}{2} \right) \cos \left(\frac{C - D}{2} \right)$
24. $\sin C - \sin D = 2 \cos \left(\frac{C + D}{2} \right) \sin \left(\frac{C - D}{2} \right)$
25. $\cos C + \cos D = 2 \cos \left(\frac{C + D}{2} \right) \cos \left(\frac{C - D}{2} \right)$
26. $\cos C - \cos D = -2 \sin \left(\frac{C + D}{2} \right) \sin \left(\frac{C - D}{2} \right)$
27. $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$
28. $\sin(A + B) - \sin(A - B) = 2 \cos A \sin B$
29. $\cos(A + B) + \cos(A - B) = 2 \cos A \cos B$

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$$30. \cos(A - B) - \cos(A + B) = 2 \sin A \sin B$$

$$31. \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R, \quad R : \text{rad. of circum circle.}$$

$$32. a^2 = b^2 + c^2 - 2bc \cos A \text{ or } \cos A = \frac{b^2 + c^2 - a^2}{2bc} \text{ similarly other 2 results.}$$

$$33. \text{Area of triangle } ABC, \Delta = \frac{1}{2} bc \sin A \text{ similarly 2 results or } \Delta = \frac{abc}{4R}$$

$$\text{or } \Delta = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{a+b+c}{2}$$

$$34. \sum n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$35. \sum n^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$36. \sum n^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2} \right)^2$$

$$37. \sinh ax = \frac{e^{ax} - e^{-ax}}{2}$$

$$38. \cosh ax = \frac{e^{ax} + e^{-ax}}{2}$$

$$39. \sin ax = \frac{e^{iax} - e^{-iax}}{2i}$$

$$40. \cos ax = \frac{e^{iax} + e^{-iax}}{2i}$$

$$41. (\cos \theta + i \sin \theta)^n = (\text{cis } \theta)^n = \text{cis } n\theta = \cos n\theta + i \sin n\theta$$

$$42. \text{Area of the circle} = \pi r^2$$

$$43. \text{Area of the ellipse} = \pi ab$$

$$44. \text{Surface area of the sphere} = 4\pi r^2$$

$$45. \text{Volume of the sphere} = \frac{4}{3} \pi r^3$$

$$46. \text{Volume of the cone} = \frac{1}{3} (\pi r^2 h)$$

$$47. \text{Volume of the right circular cylinder} = \pi r^2 h$$

$$48. \text{Total surface area of the cylinder} = \pi r (r + l) \quad (l = \text{slant height})$$

Differentiation

$$1. \frac{d}{dx} (x^n) = n \cdot x^{n-1}$$

$$2. \frac{d}{dx} (ax + b)^n = n (ax + b)^{n-1} a$$

$$3. \frac{d}{dx} (a^{mx}) = a^{mx} \log a$$

$$= m a^{mx} \log a$$

Integration

$$1. \int x^n dx = \frac{x^{n+1}}{n+1} + c \quad \text{add } c \text{ to all}$$

$$2. \int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c$$

$$3. \int a^{mx} dx = \frac{a^{mx}}{m \log a}$$

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$$4. \frac{d}{dx} (e^{mx}) = me^{mx}$$

$$4. \int e^{mx} dx = \frac{e^{mx}}{m}$$

$$5. (i) \frac{d}{dx} (\log x) = \frac{1}{x}$$

$$5. \int \frac{dx}{x} = \log x = \log_e x$$

$$(ii) \frac{d}{dx} (\log_a x) = \frac{1}{x \cdot \log a}$$

$$6. \frac{d}{dx} [\log f(x)] = \frac{f'(x)}{f(x)}$$

$$6. \int \frac{f'(x)}{f(x)} dx = \log f(x)$$

$$7. \frac{d}{dx} (\sin mx) = m \cos mx$$

$$7. \int \cos mx dx = \frac{\sin mx}{m}$$

$$8. (i) \frac{d}{dx} (\cos mx) = -m \cdot \sin mx$$

$$8. (i) \int \sin mx dx = -\frac{\cos mx}{m}$$

$$(ii) \frac{d}{dx} (\log \sec x) = \tan x$$

$$(ii) \int \tan x dx = -\log \cos x = \log \sec x$$

$$9. \frac{d}{dx} (\tan mx) = m \cdot \sec^2 mx$$

$$9. \int \sec^2 mx dx = \frac{\tan mx}{m}$$

$$10. (i) \frac{d}{dx} (\cot mx) = -m \cdot \operatorname{cosec}^2 mx$$

$$10. (i) \int \operatorname{cosec}^2 mx dx = -\frac{\cot mx}{m}$$

$$(ii) \frac{d}{dx} (\log \sin x) = \cot x$$

$$(ii) \int \cot x dx = \log \sin x$$

$$11. \frac{d}{dx} (\sec mx) = m \cdot \sec mx \cdot \tan mx$$

$$11. \int \sec mx \tan mx dx = \frac{\sec mx}{m}$$

$$12. \frac{d}{dx} (\operatorname{cosec} mx) = -m \cdot \operatorname{cosec} mx \cot mx$$

$$12. \int \operatorname{cosec} mx \cot mx dx = -\frac{\operatorname{cosec} mx}{m}$$

$$13. \frac{d}{dx} (\log (\sec x + \tan x)) = \sec x$$

$$13. \int \sec x dx = \log (\sec x + \tan x)$$

$$14. \frac{d}{dx} (\log (\operatorname{cosec} x + \cot x)) = -\operatorname{cosec} x$$

$$14. \int \operatorname{cosec} x dx = -\log (\operatorname{cosec} x + \cot x)$$

$$\text{or } = \log \left(\tan \frac{x}{2} \right)$$

$$15. \frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$15. \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x \text{ or } -\cos^{-1} x$$

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$$16. (i) \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$(ii) \frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$17. \frac{d}{dx} (\sinh x) = \cosh x$$

$$18. \frac{d}{dx} (\cosh x) = \sinh x$$

$$19. \frac{d}{dx} (\tanh x) = \operatorname{sech}^2 x$$

$$20. \frac{d}{dx} (\operatorname{cosech} x) = -\operatorname{cosech} x \cdot \coth x$$

$$21. \frac{d}{dx} (\operatorname{sech} x) = -\operatorname{sech} x \cdot \tanh x$$

$$22. \frac{d}{dx} (\coth x) = -\operatorname{cosech}^2 x$$

$$23. \frac{d}{dx} (\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$

$$24. \frac{d}{dx} (\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

$$25. \frac{d}{dx} (\tanh^{-1} x) = \frac{1}{1-x^2}$$

$$26. \frac{d}{dx} (\operatorname{cosech}^{-1} x) = -\frac{1}{x\sqrt{1+x^2}}$$

$$27. \frac{d}{dx} (\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$28. (uv)' = u'v + uv'$$

$$29. (u/v)' = \frac{u'v - uv'}{v^2}$$

$$16. (i) \int \frac{dx}{1+x^2} = \tan^{-1}(x) \text{ or } -\cot^{-1}(x)$$

$$(ii) \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x \text{ or } \operatorname{cosec}^{-1} x$$

$$17. \int \cosh x dx = \sinh x$$

$$18. \int \sinh x dx = \cosh x$$

$$19. \int \operatorname{sech}^2 x dx = \tanh x$$

$$20. \int \operatorname{cosech} x \coth x dx = -\operatorname{cosech} x$$

$$21. \int \operatorname{sech} x \cdot \tanh x dx = \operatorname{sech} x$$

$$22. \int \operatorname{cosech}^2 x dx = -\coth x$$

$$23. \int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1} x$$

$$24. \int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x$$

$$25. \int \frac{dx}{1-x^2} = \tanh^{-1} x = \frac{1}{2} \log \left(\frac{1+x}{1-x} \right)$$

$$26. \int \frac{dx}{x\sqrt{1+x^2}} = -\operatorname{cosech}^{-1} x$$

$$27. \int \frac{dx}{x\sqrt{1-x^2}} = -\operatorname{sech}^{-1} x$$

$$28. \int uv' = uv - \int u'v + \int \int v u'' + \dots$$

$$29. \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left(\frac{a+x}{a-x} \right)$$

Bernoulli's Rule

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$$30. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left(\frac{x-a}{x+a} \right)$$

$$31. \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right)$$

$$32. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$$

$$33. \int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} x$$

$$34. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

$$35. \int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$$

$$36. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a}$$

$$37. \int e^{ax} \sin(bx + c) dx = \frac{e^{ax}}{(a^2 + b^2)} [a \sin(bx + c) - b \cos(bx + c)]$$

$$\text{or} = \frac{e^{ax}}{(a^2 + b^2)^{1/2}} [(\sin(bx + c - \theta))] \text{ where } \theta = \tan^{-1}(b/a)$$

$$38. \int e^{ax} \cos(bx + c) dx = \frac{e^{ax}}{(a^2 + b^2)} [a \cos(bx + c) + b \sin(bx + c)]$$

$$\text{or} = \frac{e^{ax}}{(a^2 + b^2)^{1/2}} \cos(bx + c - \theta) \text{ where } \theta = \tan^{-1}(b/a)$$

$$39. \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$40. \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx \text{ if } f(2a-x) = f(x)$$

$$= 0 \text{ if } f(2a-x) = -f(x)$$

$$41. \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx, \text{ if } f(x) \text{ is even function, i.e., } f(-x) = f(x)$$

$$= 0 \text{ if } f(x) \text{ is odd function, i.e., } f(-x) = -f(x)$$